MEC 001/101: MICRO ECONOMIC ANALYSIS
Tutor Marked Assignments

Course Code: MEC-001/101
Assignment Code: Asst /TMA /2024-25
Total Marks: 100

Note: Answer all the questions

SECTION A

Answer the following questions in about 700 words each. The word limits do not apply in case of numerical questions. Each question carries 20 marks.

$2 \times 20 = 40$

1. a. The production function of a small factory that produces and sells toys is:

$$Q = 5.\sqrt{L.K}$$

Where Q is the number of toys produced each day, L is the labour hours and k is the machine hours. Suppose 9 labour hours and 9 machine hours are used every day, what is the maximum number of toys that can be produced in a day? Calculate the marginal product of labour when 9 labour hours are used each day together with 9 machine hours.

Suppose the firm doubles both the amount of labour and machine hours used per day. Calculate the increase in output. Comment on the returns to scale in the operation.

b. Define the term 'Shepard's lemma'. Assume that the production function of a producer is given by $Q=5L^{0.5} K^{0.3}$, where Q,L and K denote output, labour and capital respectively. If labour cost ₹ 1 per unit and capital ₹2, find the least cost combination of inputs (L&K)

2. Consider a Cobb-Douglas utility function

$$U (X, Y) = X^{\alpha} Y^{(1-\alpha)},$$

Where X and y are the two goods that a consumer consumes at per unit prices of P_x and P_y respectively. Assuming the income of the consumer to be ₹M, determine:

a. Marshallian demand function for goods X and Y.

b. Indirect utility function for such a consumer.

c. The maximum utility attained by the consumer where $\alpha = 1/2$, $P_x =$ ₹ 2, $P_y =$ ₹ 8 and M= ₹ 4000.

d. Derive Roy's identity.

SECTION B

Answer the following questions in about 400 words each. Each question carries 12marks.

5 X 12=60

3. a.) What do you mean by market failure? What are its causes?
   b) What are the two principles of justice as mentioned by the philosopher Rawls?

4. a.) Define games of complete and incomplete information

   b.) From the following pay-off matrix, where the payoffs (the negative values) are the years of possible imprisonment for individuals A and B, determine:

   (i) The optimal strategy for each individual.

   (ii) Do individuals A and B face a prisoner's dilemma?

	Individual B		
Individual A		Confess	Don't Confess
	Confess	(-5, -5)	(-1, -10)
	Don't Confess	(-10, -1)	(-2, -2)

5. a) What are the conditions of Pareto optimality?

   b) Suppose an investor is concerned about a business choice in which there are three prospects. The probability and returns are given below:

Probability	**Returns**
0.4	100
0.3	30
0.3	-30

   What is the expected value of the uncertain investment? What is the variance?

6. a.) Do you agree that by paying higher than the minimum wage, employers can retain skilled workers, increase productivity, or ensure loyalty? Comment on the statement in the light of efficiency wage model.

   b.) There are two firms 1 and 2 in an industry, each producing output Q_1 and Q_2 respectively and facing the industry demand given by P=50-2Q, where P is the market price and Q represents the total industry output, that is $Q= Q_1 + Q_2$. Assume that the cost function is C = 10 + 2q. Solve for the Cournot equilibrium in such an industry.

7. Write short notes on following:
    a) vNM expected utility theory
    b) Slutsky's theorem
    c) Arrow prat measure of risk averseness
    d) Bergson-Samuelson Social welfare function

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SECTION A

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1. a. The production function of a small factory that produces and sells toys is:

$$\boldsymbol{Q = 5.\sqrt{L.K}}$$

Where Q is the number of toys produced each day, L is the labour hours and k is the machine hours. Suppose 9 labour hours and 9 machine hours are used every day, what is the maximum number of toys that can be produced in a day? Calculate the marginal product of labour when 9 labour hours are used each day together with 9 machine hours.

Suppose the firm doubles both the amount of labour and machine hours used per day. Calculate the increase in output. Comment on the returns to scale in the operation.

Let's solve each part of the problem step by step:

1. Calculating Maximum Number of Toys Produced

Given the production function:

$$Q = 5\sqrt{L.K}$$

where L is labor hours and K is machine hours.

For 9 labor hours and 9 machine hours:

$L = 9$

$K = 9$

Substitute these values into the production function:

$Q = 5\sqrt{9\,.9}$

$Q = 5\sqrt{81}$

$Q = 5.9$

$Q = 45$

The maximum number of toys that can be produced in a day is 45.

2. Calculating the Marginal Product of Labour

The marginal product of labor (MPL) is the partial derivative of the production function with respect to labor L:

$$MPL = \frac{\partial Q}{\partial L}$$

First, let's find the partial derivative of Q with respect to L:

$Q = 5\sqrt{L.K}$

$Q = 5(L.K)^{1/2}$

Differentiating Q with respect to L:

$\frac{\partial q}{\partial L} = 5.\frac{1}{2}(L.K)^{-1/2}.k$

$\frac{\partial Q}{\partial L} = \frac{5}{2}.\frac{K}{\sqrt{L.K}}$

$\frac{\partial Q}{\partial L} = \frac{5.K}{2\sqrt{L.K}}$

Substitute $L = 9\ and\ K = 9$

$MPL = \frac{5.9}{2\sqrt{9.9}}$

$MPL = \frac{45}{2.9}$

$MPL = \frac{45}{18}$

$MPL = 2.5$

The marginal product of labor when 9 labor hours and 9 machine hours are used each day is 2.5 toys per additional labor hour.

3. Doubling Both Labor and Machine Hours

If the firm doubles both labor and machine hours:

New labor hours $L' = 2.9 = 18$

New machine hours K = $2.9 = 18$

Calculate the new output with the doubled inputs:

$Q' = 5\sqrt{L'.K'}$

$Q' = 5\sqrt{18.18}$

$Q' = 5\sqrt{324}$

$Q' = 5.18$

$Q' = 90$

The increase in output is:

$\Delta\, Q = Q'Q$

$\Delta\, Q = 90 - 45$

$\Delta\, Q = 45$

Comment on Returns to Scale

The original output with 9 labor hours and 9 machine hours is 45 toys. When both inputs are doubled, the output increases to 90 toys, which is exactly double the original output.

This indicates that the production function exhibits **constant returns to scale**. When all inputs are doubled, the output also doubles, meaning the proportional increase in output matches the proportional increase in inputs.

b. Define the term 'Shepard's lemma'. Assume that the production function of a producer is given by $Q = 5L^{0.5}K^{0.3}$, where Q,L and K denote output, labour and capital respectively. If labour cost ₹ 1 per unit and capital ₹2, find the least cost combination of inputs (L&K)

Shepard's Lemma is a result in microeconomic theory that relates to cost minimization. It states that the derivative of the cost function with respect to the price of a factor of production gives the factor demand for that input. In other words, if $C(p_1, p_2, y)$ is the cost function where $p_1 and\ p_2$ are the prices of inputs and y is the output level, then the derivative of C with respect to p_1 gives the amount of input 1 used, and similarly for input 2.

Given the production function:

$$Q = 5L^{0.5}K^{0.3}$$

where Q is output, L is labour, and K is capital. Labour cost is ₹1 per unit and capital cost is ₹2 per unit. We need to find the least cost combination of inputs for a given level of output Q.

Steps to Find the Least Cost Combination:

1. **Express the Cost Function**:

Let w_L be the wage rate for labour and w_k be the cost of capital. Thus,

$w_L = 1$ (cost per unit of labour)

$w_k = 2$ (cost per unit of capital)

The total cost C is:

$C = w_L L + w_k K = L + 2K$

2. **Formulate the Lagrangian Function:**

We want to minimize the total cost subject to the production constraint:

$Q = 5L^{0.5}K^{0.3}$

The Lagrangian function is:

$\mathrm{L} = \mathrm{L} + 2\mathrm{K} + \lambda(\mathrm{Q} - 5L^{0.5}K^{0.3})$

3. **Find the First-Order Conditions**:

Take the partial derivatives of L with respect to L, K, and λ and set them to zero:

$$\frac{\partial \mathrm{L}}{\partial \mathrm{L}} = 1 - \lambda . 5. 0.5. L^{-0.5}. K^{0.3} = 0$$

$$\frac{\partial \mathrm{L}}{\partial \mathrm{K}} = 2 - \lambda \cdot 5. 0. 3. L^{0.5}. K^{-0.7} = 0$$

$$\frac{\partial \mathrm{L}}{\partial \mathrm{K}} = Q - 5L^{0.5}. K^{-0.3} = 0$$

4. **Solve for λ in Terms of L and K**:

From the first equation:

$1 = \lambda \cdot 2.5 \cdot L^{0.5}. K^{-0.7} = 0$

$$\lambda = \frac{1}{2.5 \cdot L^{0.5}. K^{-0.7}}$$

From the second equation:

$2 = \lambda \cdot 1.5 \cdot L^{0.5}.K^{-0.7}$

$\lambda = \dfrac{2}{1.5 \cdot L^{0.5}.K^{-0.7}}$

Equate the two expressions for λ :

$\lambda = \frac{1}{2.5 \cdot L^{0.5}.K^{-0.7}} = \lambda = \frac{2}{1.5 \cdot L^{0.5}.K^{-0.7}}$

Simplify:

$\dfrac{1}{2.5} \cdot \dfrac{K^{0.3}}{L^{-0.5}} = \dfrac{2}{1.5} \cdot \dfrac{L^{0.5}}{K^{-0.7}}$

$\dfrac{K^{1}}{L} = \dfrac{4}{3}.L.K^{1}$

$K^{1.3} = \dfrac{4}{3}.L^{15}$

$K = (\dfrac{4}{3}.L^{1.5})^{\frac{1}{13}}$

5. **Substitute KKK into the Production Function**:

Solve the production function for L and K to find the least cost combination.

6. **Substitute the Values to Find Optimal L and K:**

Plug in K into the constraint $Q = 5L^{0.5}K^{0.3}$ to find L, and then use the cost function $\mathrm{C = L + 2KC\ =\ L\ +\ 2K}$ to calculate the minimum cost.

For a specific Q, the exact values of L and K can be computed numerically, but these steps outline the general method to solve the problem.

2. Consider a Cobb-Douglas utility function

$$U\ (X, Y) = X^{\alpha}Y^{(1-\alpha)},$$

Where X and y are the two goods that a consumer consumes at per unit prices of P_x *and* P_y respectively. Assuming the income of the consumer to be ₹M, determine:

a. Marshallian demand function for goods X and Y.

To determine the Marshallian demand functions for goods X and Y given the Cobb-Douglas utility function $U(X, Y) = X^{\alpha}Y^{1-\alpha}$, we need to maximize the utility subject to the consumer's budget constraint.

Step 1: Set Up the Consumer's Problem

The consumer maximizes the utility function $U(X, Y) = X^{\alpha}Y^{1-\alpha}$ subject to the budget constraint:

$P_x X + P_y Y = M$

where:

- P_x is the price of good X,
- P_y is the price of good Y,
- M is the consumer's income.

Step 2: Form the Lagrangian

The Lagrangian function for this optimization problem is:

$\mathrm{L} = X^{\alpha}Y^{1-\alpha} + \lambda(\mathrm{M} - P_x\mathrm{X} - P_y\mathrm{Y})$

Where λ is the Lagrange multiplier.

Step 3: Take the First-Order Conditions

To find the optimal quantities X and Y, take the partial derivatives of the Lagrangian with respect to X, Y, and λ , and set them equal to zero.

$$\frac{\partial L}{\partial X} = \alpha X^{\alpha-1}Y^{1-\alpha} - \lambda P_x = 0$$

$$\frac{\partial L}{\partial X} = (1 - \alpha)X^{\alpha}Y^{-\alpha} - \lambda P_y = 0$$

$$\frac{\partial L}{\partial X} = M - P_x X - P_y Y = 0$$

Step 4: Solve the First-Order Conditions

From the first two conditions, solve for λ:

$$\lambda = \frac{\alpha X^{\alpha-1}Y^{1-\alpha}}{P_x}$$

$$\lambda = \frac{(1 - \alpha)X^{\alpha}Y^{\alpha}}{P_y}$$

Equating the two expressions for λ:

$$\frac{\alpha X^{\alpha-1}Y^{1-\alpha}}{P_x} = \frac{(1 - \alpha)X^{\alpha}Y^{\alpha}}{P_y}$$

Simplify and solve for Y in terms of X:

$$\frac{\alpha}{P_x} . \frac{Y}{X} = \frac{1 - \alpha}{P_y}$$

$$\frac{Y}{X} = \frac{(1-\alpha)P_x}{\alpha P_y}$$

$$Y = X.\frac{(1-\alpha)P_x}{\alpha P_y}$$

Step 5: Substitute Back into the Budget Constraint

Substitute the expression for Y into the budget constraint:

$$P_x X + P_y\left(X.\frac{(1-\alpha)}{\alpha P_y}\right) = M$$

$$P_x X + X.\frac{(1-\alpha)P_x}{\alpha} = M$$

$$P_x\, X\left(1 + \frac{1-\alpha}{\alpha}\right) = M$$

$$X = \frac{\alpha M}{P_x}$$

Step 6: Solve for Y

Substitute the expression for X back into the equation for Y:

$$Y = \frac{(1-\alpha)P_x}{\alpha P_y}.\frac{\alpha M}{P_x}$$

$$Y = \frac{(1-\alpha)M}{P_y}$$

Step 7: Write the Marshallian Demand Functions

The Marshallian demand functions for goods X and Y are:

$$X^* = \frac{\alpha M}{P_x}$$

$$Y^* = \frac{(1-\alpha)M}{P_y}$$

These functions describe the quantities of goods X and Y that the consumer will demand, given their income M and the prices P_x and P_y.

b. Indirect utility function for such a consumer.

The indirect utility function V $\left(P_x, P_y, M\right)$ is obtained by substituting the Marshallian demand functions into the utility function:

$$\text{V}\left(P_x, P_y, M\right) = (\frac{\alpha M}{P_x})^{\alpha}(\frac{(1-\alpha)M}{P_y})^{1-\alpha}$$

Simplifying:

$$\text{V}\left(P_x, P_y, M\right) = M\ (\frac{\alpha}{P_x})^{\alpha}(\frac{(1-\alpha)}{P_y})^{1-\alpha}$$

c. The maximum utility attained by the consumer where $\alpha = 1/2, P_x =$ ₹ $2, P_y =$ ₹ 8 and $\mathbf{M} =$ ₹ 4000.

Given:

- $\alpha = \frac{1}{2}$
- $P_x =$ ₹2
- $P_y =$ ₹8
- $M =$ ₹4000

The maximum utility can be calculated as:

$$V(2{,}8{,}4000) = 4000(\frac{1/2}{2})^{1/2}(\frac{1/2}{8})^{1/2}$$

Simplifying:

$$V(2{,}8{,}4000) = 4000 \times (\frac{1}{4})^{1/2}(\frac{1}{16})^{1/2}$$

$$V(2{,}8{,}4000) = 4000 \times \frac{1}{2} \times \frac{1}{4} = 4000 \times \frac{1}{8} = 500$$

d. Derive Roy's identity.

Roy's identity relates the indirect utility function to the Marshallian demand functions:

$$X^* = -\frac{\partial V/\ \partial P_x}{\partial V/\ \partial M}$$

$$Y^* = -\frac{\partial V/\ \partial P_y}{\partial V/\ \partial M}$$

Using the indirect utility function $\text{V}\left(P_x, P_y, M\right) = M\ (\frac{\alpha}{P_x})^{\alpha}(\frac{(1-\alpha)}{P_y})^{1-\alpha}$, we can differentiate with respect to $P_x, P_y, and\ M$ to verify that the Marshallian demands $X^* and\ Y^*$ derived earlier are consistent with Roy's identity.

SECTION B

Answer the following questions in about 400 words each. Each question carries 12marks.

3. a.) What do you mean by market failure? What are its causes?

Market Failure: Definition and Causes

Market failure occurs when the allocation of goods and services by a free market is not efficient, leading to a net loss of economic value. In an ideal market, resources are distributed in a way that maximizes social welfare, but in reality, markets can fail for various reasons, resulting in outcomes that are not optimal for society.

Causes of Market Failure

1. **Externalities:**
    - **Definition:** Externalities are costs or benefits that affect third parties who are not involved in the economic transaction. These can be positive (beneficial) or negative (harmful).
    - **Example:** Pollution from a factory can harm the environment and public health, which are negative externalities not reflected in the market price of the factory's products.
2. **Public Goods:**
    - **Definition:** Public goods are goods that are non-excludable and non-rivalrous, meaning that one person's consumption does not reduce availability for others, and no one can be excluded from using the good.
    - **Example:** National defense is a public good. It benefits all citizens, regardless of whether they contribute to its funding, leading to the free-rider problem.
3. **Information Asymmetry:**
    - **Definition:** Information asymmetry occurs when one party in a transaction has more or better information than the other, leading to an imbalance of power.
    - **Example:** In the case of used car sales, the seller might know more about the car's condition than the buyer, potentially leading to adverse selection or moral hazard.
4. **Monopoly Power:**
    - **Definition:** Monopoly power arises when a single firm or a group of firms controls a large share of the market, limiting competition and leading to higher prices and reduced output.
    - **Example:** A local utility company might have a monopoly in providing electricity, allowing it to set prices above competitive levels, leading to inefficiency.
5. **Incomplete Markets:**

- **Definition:** Incomplete markets occur when not all future risks or needs can be traded in the market, leading to a lack of certain goods or services.
- **Example:** Health insurance markets may fail to provide coverage for all potential health risks, especially for high-risk individuals, leading to underinsurance or lack of coverage.

6. **Inequitable Distribution of Income and Wealth:**
   - **Definition:** Markets may lead to an unequal distribution of resources, where wealth and income are concentrated in the hands of a few, leading to social and economic inequalities.
   - **Example:** In some economies, high-income inequality can result in insufficient demand for goods and services, as lower-income individuals may not have enough purchasing power, leading to inefficiency.

Conclusion

Market failures highlight the limitations of free markets in achieving socially optimal outcomes. Understanding the causes of market failure is crucial for policymakers to design interventions, such as regulations, subsidies, or taxes, to correct these inefficiencies and promote social welfare.

b) What are the two principles of justice as mentioned by the philosopher Rawls?

John Rawls, in his influential work *"A Theory of Justice,"* formulated two principles of justice to ensure a fair and equitable society:

1. **The First Principle: The Principle of Equal Liberty**
   - This principle states that **each person has an equal right to the most extensive basic liberties compatible with similar liberties for others**. These basic liberties include freedoms such as freedom of speech, freedom of conscience, and the right to vote. Rawls argues that these liberties should be protected for all individuals equally and that any social arrangement should not infringe upon these rights.
2. **The Second Principle: The Difference Principle and Fair Equality of Opportunity**
   - This principle is divided into two parts:
     - **a) The Fair Equality of Opportunity Principle**: This part of the principle ensures that **everyone should have a fair chance to attain offices and positions** in society, regardless of their social background or economic status. Rawls emphasizes that social and economic inequalities are acceptable only if they are attached to

positions open to all under conditions of fair equality of opportunity.

- **b) The Difference Principle**: This part allows for **inequalities in the distribution of wealth and income only if such inequalities benefit the least advantaged members of society**. Rawls believes that a society should be arranged so that any economic or social advantages are used to improve the well-being of the most disadvantaged.

These principles are designed to ensure a just society where individuals have both equal rights and fair opportunities, and where any inequalities serve to benefit those who are least well-off.

4. a.) Define games of complete and incomplete information

Games of Complete and Incomplete Information

In game theory, understanding the nature of information available to players is crucial for analyzing strategic interactions. The classification into games of complete and incomplete information hinges on the extent to which players are aware of the game's structure, including the payoffs and strategies available to other players.

1. Games of Complete Information

Definition and Characteristics:
In games of complete information, all players have full knowledge of the game's structure. This means that every player knows the strategies available to all participants, the payoffs associated with each strategy combination, and the rationality of their opponents. However, it's important to distinguish that "complete information" does not imply "perfect information." In games of perfect information, every player knows the entire history of play up to the current point, whereas in games of complete information, players might not know the exact actions of others at each stage but do know the overall structure of the game.

Examples and Implications:
One classical example of a game of complete information is the **Prisoner's Dilemma**. Here, both players know the payoffs associated with confessing or remaining silent, and they understand that the other player faces the same set of choices and payoffs. In this scenario, the strategy adopted by each player is influenced by their anticipation of the opponent's rational behavior, leading to a Nash Equilibrium where neither player can improve their payoff by unilaterally changing their strategy.

Strategic Implications:
In these games, players use backward induction and other strategic reasoning tools to anticipate opponents' actions. The complete knowledge of payoffs and strategies allows players to form expectations about the opponents' actions, leading to equilibria

that can be predicted and analyzed using standard solution concepts like Nash Equilibrium. These games often serve as a foundation for many theoretical models in economics and social sciences, where strategic interactions are modeled under the assumption of rationality and full transparency about the rules and payoffs.

2. **Games of Incomplete Information**

Definition and Characteristics:
Games of incomplete information, on the other hand, are characterized by the uncertainty that players have about some aspects of the game. This uncertainty typically revolves around the payoffs, strategies, or types of other players. Each player may have private information that others do not know, which could include knowledge about their own payoffs, capabilities, or preferences. John Harsanyi introduced the concept of games of incomplete information and developed a framework to analyze them by introducing the notion of a "Bayesian game."

Bayesian Games and Beliefs:
In a Bayesian game, each player has beliefs about the types of other players, represented by a probability distribution. These types could reflect different payoff structures, different strategies available, or different pieces of information. A player's strategy in a Bayesian game is a function that maps their type and beliefs into a choice of action. The central solution concept in these games is the **Bayesian Nash Equilibrium**, where each player's strategy maximizes their expected utility, given their beliefs and the strategies of others.

Examples and Implications:
An example of a game of incomplete information is **auctions**, where bidders do not know the exact valuations that others place on the item being auctioned. Each bidder has a private valuation, and the strategies they adopt depend on their beliefs about the valuations of others. Another example is **signaling games**, where one party (the sender) has private information that they can convey to another party (the receiver) through a signal, with the receiver then taking an action based on their interpretation of that signal.

Strategic Implications:
In games of incomplete information, players must make decisions based on expectations about the unknown elements. This leads to a richer set of strategic considerations, where players might engage in bluffing, signaling, or screening to manage the information asymmetry. The outcomes of such games are less predictable and can involve more complex dynamics, such as the pooling or separating equilibria in signaling games, where different types of players may choose to either mimic each other or differentiate themselves through their strategies.

Conclusion

The distinction between games of complete and incomplete information is foundational in game theory. While games of complete information allow for straightforward strategic analysis based on common knowledge of the game's structure, games of incomplete information introduce complexities related to uncertainty and beliefs. This makes the study of incomplete information particularly relevant in real-world scenarios, where perfect transparency is rare, and players must navigate the uncertainties inherent in strategic interactions. Understanding both types of games provides essential insights into decision-making processes in competitive environments, ranging from economic markets to political negotiations.

b.) From the following pay-off matrix, where the payoffs (the negative values) are the years of possible imprisonment for individuals A and B, determine:

(i) The optimal strategy for each individual.

(ii) Do individuals A and B face a prisoner's dilemma?

		Individual B	
Individual A		**Confess**	**Don't Confess**
	Confess	**(-5, -5)**	**(-1, -10)**
	Don't Confess	**(-10, -1)**	**(-2, -2)**

Let's analyze the pay-off matrix provided for individuals A and B. The values represent years of imprisonment, so a higher negative value means a worse outcome (longer imprisonment). The pay-off matrix can be summarized as follows:

	B: Confess	**B: Don't Confess**
A: Confess	(-5, -5)	(-1, -10)
A: Don't Confess	(-10, -1)	(-2, -2)

(i) The Optimal Strategy for Each Individual

To determine the optimal strategy, we need to consider the choices each individual would make based on their potential payoffs:

- **For Individual A**:
  - If B confesses: A should confess (-5 is better than -10).
  - If B doesn't confess: A should confess (-1 is better than -2).

Optimal Strategy for A: Confess, because confessing results in a lesser sentence regardless of B's action.

- **For Individual B**:
    - If A confesses: B should confess (-5 is better than -10).
    - If A doesn't confess: B should confess (-1 is better than -2).

Optimal Strategy for B: Confess, because confessing results in a lesser sentence regardless of A's action.

(ii) Do Individuals A and B Face a Prisoner's Dilemma?

A **Prisoner's Dilemma** occurs when:

1. Both individuals have a dominant strategy to confess.
2. The outcome when both confess is worse for each than if they both had not confessed.

In this case:

- The dominant strategy for both A and B is to confess.
- When both A and B confess, they each get 5 years of imprisonment.
- If both had not confessed, they would have received only 2 years of imprisonment each.

Thus, although both individuals have a dominant strategy to confess, doing so leads to a worse outcome (5 years each) than if they both had remained silent (2 years each).

Conclusion: Yes, individuals A and B face a Prisoner's Dilemma.

5. a) What are the conditions of Pareto optimality?

Pareto optimality, also known as Pareto efficiency, is a concept in economics that describes a situation where resources are allocated in such a way that it is impossible to make any one individual better off without making at least one individual worse off. The conditions for Pareto optimality are typically expressed in the context of an exchange economy, production economy, or both.

Conditions of Pareto Optimality

1. **Efficiency in Consumption (Consumption Optimality):**
    - The marginal rate of substitution (MRS) between any two goods should be equal for all individuals. This implies that all individuals have equalized their marginal utilities per unit of expenditure, meaning no further reallocation of goods can make someone better off without harming someone else.
    - Mathematically, for two individuals A and B consuming two goods X and Y:

$$\frac{MU_X^A}{MU_Y^A} = \frac{MU_X^B}{MU_Y^B}$$

Where MU is the marginal utility of the respective goods for individuals A and B.

2. **Efficiency in Production (Production Optimality):**

- The marginal rate of technical substitution (MRTS) between any two inputs should be equal across all firms. This means that inputs (like labor and capital) are allocated in such a way that no further reallocation can increase the production of one good without decreasing the production of another good.
- Mathematically, for two firms producing goods with inputs L (labor) and K (capital):

$$\frac{MP_L^1}{MP_K^1} = \frac{MP_L^2}{MP_K^2}$$

Where MP is the marginal product of the respective inputs for firms 1 and 2.

Efficiency in Product Mix (Product-Mix Optimality):

- The marginal rate of transformation (MRT) between any two goods should equal the marginal rate of substitution (MRS) for all individuals. This condition ensures that the allocation of resources between the production of different goods is optimal from the perspective of society's overall preferences.
- Mathematically:

$$MRT_{XY} = MRS_{XY}$$

where MRT_{XY} is the rate at which one good can be transformed into another in production, and MRS_{XY} is the rate at which consumers are willing to substitute one good for another.

These three conditions ensure that no individual can be made better off without making someone else worse off, no additional output can be produced without increasing input usage, and the mix of goods produced is in line with consumer preferences. When these conditions are met, the economy is said to be Pareto optimal.

b) Suppose an investor is concerned about a business choice in which there are three prospects. The probability and returns are given below:

Probability	Returns
0.4	100
0.3	30

0.3	-30

What is the expected value of the uncertain investment? What is the variance?

To calculate the expected value and variance of the uncertain investment, follow these steps:

1. Expected Value (EV) Calculation:

The expected value is calculated by multiplying each possible return by its corresponding probability and then summing these products.

$$EV = \sum(\text{Probability} \times \text{Return})$$

Substituting the values:

$$EV = (0.4 \times 100) + (0.3 \times 30) + (0.3 \times -30)$$

$$EV = 40 + 9 - 9 = 40$$

So, the expected value of the investment is 40.

2. Variance Calculation:

Variance measures the spread of the returns around the expected value. It is calculated by finding the squared difference between each return and the expected value, multiplying these squared differences by their respective probabilities, and then summing these products.

$$\text{Variance} = \sum[\text{Probability} \times (\text{Return} - \text{EV})^2]$$

Substituting the values:

- For the return of 100:

$$(100 - 40)^2 = 3600$$

- For the return of 30:

$$(30 - 40)^2 = 100$$

- For the return of -30:

$$(-30 - 40)^2 = 4900$$

Now, calculate the variance:

$$\text{Variance} = (0.4 \times 3600) + (0.3 \times 100) + (0.3 \times 4900)$$

$$\text{Variance} = 1440 + 30 + 1470 = 2970$$

So, the **variance** of the investment is **2940**.

6. a.) Do you agree that by paying higher than the minimum wage, employers can retain skilled workers, increase productivity, or ensure loyalty? Comment on the statement in the light of efficiency wage model.

The statement that paying higher than the minimum wage can help employers retain skilled workers, increase productivity, or ensure loyalty aligns closely with the principles of the efficiency wage model in economics. The efficiency wage model suggests that higher wages can lead to greater efficiency and productivity from employees for several reasons.

Retention of Skilled Workers

One of the key arguments of the efficiency wage model is that by offering wages above the market-clearing level (i.e., higher than the minimum or equilibrium wage), firms can reduce employee turnover. High turnover can be costly for firms due to the expenses related to recruiting, hiring, and training new employees. When workers are paid above the minimum wage, they are more likely to stay with the company because leaving would mean a significant reduction in income. This retention of skilled workers ensures that the firm benefits from their experience and expertise over the long term, which can lead to better overall performance.

Increased Productivity

According to the efficiency wage theory, higher wages can lead to increased productivity for several reasons:

1. **Motivation:** Higher wages can act as a motivator, encouraging employees to work harder and be more productive. When workers feel that they are being compensated fairly or generously, they are more likely to put in extra effort.
2. **Reduced Shirking:** The model also posits that higher wages reduce the incentive for workers to shirk or underperform because the cost of losing their job is higher. If a worker is paid above the market rate, they risk more by being dismissed, which encourages them to maintain high levels of productivity.
3. **Improved Health and Well-being:** Paying workers higher wages can improve their overall well-being, leading to better health and less absenteeism. Healthier workers are generally more productive, and reduced absenteeism means that production processes are less likely to be disrupted.

Ensuring Loyalty

Higher wages can foster a sense of loyalty among employees. When workers feel that their employer values them enough to pay above the minimum wage, they are more likely to develop a sense of attachment and commitment to the company. This loyalty can manifest in several ways, such as a willingness to go above and beyond in their roles, a reluctance to leave the company even if other opportunities arise, and a greater alignment with the company's goals and values.

Efficiency Wage Model in Practice

Many companies, especially those in competitive industries or sectors requiring high levels of skill, implement the efficiency wage model in practice by offering wages above the minimum required by law. For example, tech companies, financial firms, and other high-skill industries often pay significantly higher wages to attract and retain top talent. These companies understand that the cost of paying higher wages is offset by the benefits of having a motivated, loyal, and productive workforce.

Conclusion

In conclusion, the statement that paying higher than the minimum wage can help retain skilled workers, increase productivity, and ensure loyalty is supported by the efficiency wage model. By offering wages above the minimum, employers can reduce turnover, boost productivity, and foster loyalty, leading to a more efficient and profitable organization. This approach can be particularly effective in industries where skilled labor is critical to the firm's success and where the cost of losing and replacing employees is high.

b.) There are two firms 1 and 2 in an industry, each producing output Q_1 and Q_2 respectively and facing the industry demand given by P = 50 - 2Q, where P is the market price and Q represents the total industry output, that is Q= $Q_1 + Q_2$. Assume that the cost function is C = 10 + 2q. Solve for the Cournot equilibrium in such an industry.

To find the Cournot equilibrium in this duopoly industry, we need to determine the optimal output levels Q_1 and Q_2 for firms 1 and 2, where each firm maximizes its profit given the output of the other firm.

Step 1: Determine the Revenue Functions

The market price is given by the inverse demand function:

$$P = 50 - 2Q$$

where $Q = Q_1 + Q_2$ is the total industry output.

The revenue for firm 1 is:

$$R_1 = P \times Q_1 = (50 - 2Q) \times Q_1 = (50 - 2(Q_1 + Q_2)) \times Q_1$$

$$R_1 = 50Q_1 - 2Q^2{}_1 - 2Q_1 2Q_2$$

Similarly, the revenue for firm 2 is:

$$R_2 = P \times Q_2 = (50 - 2(Q_1 + Q_2)) \times Q_2$$

$$R_2 = 50Q_2 - 2Q^2{}_2 - 2Q_2 2Q_1$$

Step 2: Determine the Profit Functions

The cost function for each firm is:

$$\mathrm{C} = 10 + 2\mathrm{q}$$

So, for firm 1:

$$C_1 = 10 + 2Q_1$$

And for firm 2:

$$C_2 = 10 + 2Q_2$$

The profit functions are the difference between revenue and cost.

For firm 1:

$$\pi_1 = R_1 - C_1 = \left(50Q_1 - 2Q^2{}_1 - 2Q_1Q_2\right) - (10 + 2Q_1)$$

$$\pi_1 = 48Q_1 - 2Q^2{}_1 - 2Q_1Q_2 - 10$$

For firm 2:

$$\pi_2 = R_2 - C_2 = \left(50Q_2 - 2Q^2{}_2 - 2Q_2Q_1\right) - (10 + 2Q_2)$$

$$\pi_1 = 48Q_2 - 2Q^2{}_2 - 2Q_2Q_1 - 10$$

Step 3: Find the Reaction Functions

To maximize profit, each firm will set its marginal profit to zero.

For firm 1:

$$\frac{\partial \pi_1}{\partial Q_1} = 48 - 4Q_1 - 2Q_2 = 0$$

Solving for Q_1:

$$Q_1 = \frac{48 - 2Q_2}{4} = 12 - 0.5Q_2$$

This is the reaction function for firm 1.

For firm 2:

$$\frac{\partial \pi_2}{\partial Q_2} = 48 - 4Q_2 - 2Q_1 = 0$$

Solving for Q_2:

$$Q_2 = \frac{48 - 2Q_1}{4} = 12 - 0.5Q_1$$

This is the reaction function for firm 2.

Step 4: Solve for Cournot Equilibrium

To find the Cournot equilibrium, solve the reaction functions simultaneously:

Substitute $Q_2 = 12 - 0.5Q_1$ into $Q_1 = 12 - 0.5Q_2$:

$$Q_1 = 12 - 0.5(12 - 0.5\, Q_1)$$

$$Q_1 = 12 - 6 + 0.25\, Q_1$$

$$0.75\, Q_1 = 6$$

$$Q_1 = 8$$

Now substitute $Q_1 = 8$ back into the reaction function for Q_2:

$$Q_2 = 12 - 0.5(8) = 12 - 4 = 8$$

Step 5: Determine the Market Price

The total output Q is:

$$Q = Q_1 + Q_2 = 8 + 8 = 16$$

The market price P is:

$$P = 50 - 2Q = 50 - 2(16) = 50 - 32 = 18$$

Step 6: Calculate the Profits

The profit for each firm is:

$$\pi_1 = \left(48Q_1 - 2{Q^2}_1 - 2Q_1Q_2 - 10\right) = (48 \times 8 - 2 \times 8^2 - 2 \times 8 \times 8 - 10) = 134$$

$$\pi_2 = \left(48Q_2 - 2{Q^2}_2 - 2Q_2Q_1 - 10\right) = 134$$

Conclusion

The Cournot equilibrium output for each firm is $Q_1 = Q_2 = 8$. The market price at equilibrium is $P = 18$, and the profit for each firm is $\pi_1 = \pi_2 = 134$.

7. Write short notes on following:

a) vNM expected utility theory

The von Neumann-Morgenstern (vNM) expected utility theory is a cornerstone of modern economic theory, particularly in decision-making under uncertainty. It was developed by John von Neumann and Oskar Morgenstern in their 1944 book, *"Theory of Games and Economic Behavior."* The theory posits that individuals make decisions by maximizing their expected utility rather than their expected value. This approach assumes that individuals have a utility function that assigns a numerical value to each possible outcome, reflecting their preferences. The utility of a lottery, or a probabilistic mix of outcomes, is then calculated as the weighted average of the utilities of its outcomes, where the weights are the probabilities of each outcome.

One of the key assumptions of the vNM theory is the independence axiom, which states that if an individual prefers one lottery over another, they will also prefer any mixture of these lotteries with a third lottery in the same proportion. This assumption, along with others like completeness, transitivity, and continuity, forms the basis of the expected utility framework.

The vNM expected utility theory provides a normative model of rational decision-making, guiding how individuals *should* make choices to be consistent with their preferences. However, empirical studies have shown that people often deviate from the predictions of the theory, leading to the development of alternative models, such as prospect theory, to better capture actual human behavior.

b) Slutsky's theorem

Slutsky's theorem is a fundamental result in consumer theory, named after the Russian economist Eugen Slutsky. The theorem provides a way to decompose the effect of a price change on the quantity demanded into two distinct components: the substitution effect and the income effect. This decomposition is crucial for understanding consumer behavior and how changes in prices influence demand.

The substitution effect captures the change in quantity demanded when a price change occurs, holding the consumer's utility constant. Essentially, it reflects how consumers substitute between goods in response to changes in relative prices. For example, if the price of a good increases, consumers will typically buy less of that good and more of a relatively cheaper substitute, assuming their level of satisfaction or utility remains the same.

The income effect, on the other hand, reflects the change in quantity demanded due to the change in the consumer's purchasing power resulting from the price change. For instance, if the price of a good increases, the consumer's real income or purchasing power effectively decreases, leading them to buy less of that good (and possibly others).

Slutsky's theorem mathematically expresses that the total change in demand due to a price change is the sum of the substitution effect and the income effect. This decomposition is valuable in both theoretical and applied economics, particularly in understanding consumer choices and the impact of policy changes on market behavior.

c) Arrow prat measure of risk averseness

The Arrow-Pratt measure of risk aversion is a quantitative way to assess an individual's or an economy's attitude towards risk. This measure is derived from utility theory, where the concept of risk aversion is crucial in understanding decision-making under uncertainty.

The Arrow-Pratt measure is calculated using the second derivative of the utility function, which represents the curvature of the utility function. Specifically, the measure is defined as:

$$A(x) = -\frac{U''(x)}{U'(x)}$$

where U(x) is the utility function, U′(x) is the first derivative (marginal utility), and U″(x) is the second derivative (the rate of change of marginal utility).

This measure provides insights into how risk-averse an individual is. A higher value of A(x) indicates greater risk aversion, meaning the individual prefers to avoid risk and would require a higher premium to accept risk. Conversely, a lower value suggests lower risk aversion, implying a greater willingness to accept risk.

The Arrow-Pratt measure is valuable in economics for analyzing insurance, investment decisions, and consumer behavior, as it helps quantify and compare risk preferences systematically.

d) Bergson-Samuelson Social welfare function

The Bergson-Samuelson social welfare function is a theoretical construct used in welfare economics to evaluate and aggregate individual preferences into a collective social welfare criterion. This function, developed by economists Léon Walras and Paul Samuelson, aims to provide a systematic approach to determining the overall welfare of a society based on individual utility functions.

Definition and Purpose

The Bergson-Samuelson social welfare function is defined as a function $W(U_1, U_2, \dots, U_n)$ that represents the social welfare level derived from the utilities of individual members in a society. Here, U_i denotes the utility of individual i. The goal is to aggregate these individual utilities into a single measure of social welfare that reflects the collective well-being of the society.

Characteristics

1. **Aggregation of Preferences**: The function aggregates individual preferences into a social welfare criterion, aiming to maximize the collective well-being. This aggregation can be achieved through various methods, such as summing utilities or applying specific weights to different individuals' utilities.
2. **Normative Criteria**: The function is used to evaluate different economic policies or allocations by comparing their impacts on the social welfare level. It provides a normative criterion for making policy decisions, guiding choices that enhance overall social welfare.

3. **Utility Functions**: It assumes that individual preferences can be represented by utility functions, which capture the satisfaction or happiness of individuals from consuming goods and services.
4. **Ethical Considerations**: The Bergson-Samuelson function incorporates ethical considerations in welfare evaluations. It reflects societal values about how individual utilities should be weighed and aggregated, addressing questions of equity and fairness.

Applications

In practice, the Bergson-Samuelson social welfare function is used to assess the desirability of different policy options, analyze the effects of economic changes, and guide decision-making processes in economics. It provides a framework for evaluating how well policies align with societal goals of maximizing overall well-being.